Critical Communication Radius for Sink Connectivity in Wireless Networks

Hongchao Zhou, Fei Liu, Xiaohong Guan

Tsinghua University / Xi’an Jiaotong University
Outlines

- Introduction
- Asymptotic sink connectivity
- Critical communication radius for sink connectivity
- Effective communication radiiuses for different link models
Wireless Sensor Networks

- Small devices with capability of sensing, processing and wireless communication
- Distributed and autonomous wireless networks with self-organization and cooperation for information acquisition
- Wide variety of applications for infrastructure safety, environmental monitoring, manufacturing and production, logistics, health care, security surveillance, target detection/localization/tracking, etc
Challenging problems and issues

- Limited node resources in terms of energy, bandwidth, processing capacity, storage, etc.
- Energy consumption $\propto \{\text{processing speed}^{2-4}, \text{sensing radius}^{q=2-4}, \text{communication radius}^{q=2-4}\}$
- Energy constrained communication protocol
- Special issues on connectivity, time synchronization, localization, sensing coverage, task allocation, data management, etc.
Determine the minimal $r$ to guarantee the connectivity of the network $G(n, s, r)$: the network in consideration

$s$: disc radius

$A$: disc area, $A = \pi s^2$

$r$: communication radius, if $\|x_i - x_j\| < r$, $i \rightarrow j$ and $j \rightarrow i$

$n$: the number of nodes

$d$: $d = n \pi r^2 / A$, average number of neighbor nodes

Connectivity problem

Arnd $\frac{r}{2} = \frac{2sA}{\pi}$
Existing result
(P. Gupta and P. R. Kumar, 1998)

- Critical radius for fully connected graph (no isolated node)

The network is asymptotically \((n \to \infty)\) fully connected with probability one if and only if

\[
r = \sqrt{\frac{A(\log n + \gamma)}{\pi n}}
\]

with variable \(\gamma \to \infty\)
Issue

- Full connection may not be necessary for some applications
- To save energy and prolong lifetime, a very small fraction isolated nodes of in a wireless sensor network with thousands of nodes could be tolerated
Introducing sink connectivity

- Assume the sink is a randomly selected node in the network
- Sink connectivity $C_n$ is defined as the fraction of nodes in the network that are connected to the sink
Goal

- Find the critical communication radius to guarantee $C_n > \alpha$, where $\alpha$ is a constant close to 1.
Let $L_j(n,s,r)$ be the number of nodes in the $j$th-largest connected subnet in $G(n,s,r)$.
Fully connected

\[ \iff C_n = 1 \]

\[ \iff L_1(n, s, r) = n \]
Partial connected

\[ \Leftrightarrow C_n < 1 \]

\[ \Leftrightarrow L_1(n, s, r) < n \]

The expectation of \( C_n \)

\[ E(C_n) = \sum_i \left( \frac{L_i(n, s, r)}{n} \right)^2 \]
Outlines

- Introduction
- Asymptotic sink connectivity
- Critical communication radius for sink connectivity
- Effective communication radiiuses for different link models
Asymptotic sink connectivity

Based on the continuum percolation theory*, we can get the following two theorems

**Theorem 1:** If \( r_n = \sqrt{Ad/(\pi n)} \), let \( d_c = \pi \lambda_c \), then \( \forall d < d_c, C_n \xrightarrow{P} 0 \) as \( n \to \infty \).

**Theorem 3:** If \( r_n = \sqrt{Ad/(\pi n)} \), then, \( C_n \xrightarrow{P} 1 \) as \( n \to \infty \) if and only if \( d \to \infty \).

Comparison with the existing result

Gupta’s conclusion

Goal

\[ P_c = \mathbb{P}[C_n = 1] \rightarrow 1 \quad \text{as} \quad n \rightarrow \infty \]

Critical radius:

\[ r^* = \sqrt{\frac{A(\log n + \gamma)}{\pi n}} \]

where \( \gamma \rightarrow \infty \)

Example:

\[ r^* = \sqrt{\frac{A(\log n + \log \log n)}{\pi n}} \]

Current result

Goal

\[ C_n \xrightarrow{\mathbb{P}} 1 \quad \text{as} \quad n \rightarrow \infty \]

Requirement:

\[ r = \sqrt{\frac{A}{d/(\pi n)}} \]

where \( d \rightarrow \infty \)

Example:

\[ r = \sqrt{\frac{A}{(\log \log n)/(\pi n)}} \]
Average neighbor number $d$

- $d = \frac{n \pi r^2}{A}$

- **Mapping**: $G(n, s, r) \xrightarrow{\times \lambda} G(n, \lambda s, \lambda r)$
  The connectivity is unchanged; $d = \frac{n \pi r^2}{A}$ is unchanged.

- Instead of $r$, we discuss the relationship between the connectivity and $d$ for simplify.

- Using $r = \sqrt{\frac{Ad}{\pi n}}$, we can get the corresponding communication radius.
Connectivity versus average number of neighbors

Fig. 2. relations between $C_n$, $P_c$ and $d$. 

$P_c = \mathbb{P}[C_n = 1]$
Outlines

- Introduction
- Asymptotic sink connectivity
- Critical communication radius for sink connectivity
- Effective communication radiuses for different link models
A network is $\alpha$ sink connected if $C_n \geq \alpha$ with high probability.

The minimal radius that makes the network $\alpha$ sink connected is the critical communication radius for $\alpha$ sink connected.
Required average neighbor number versus $n$

Critical radius

\[ r = \sqrt{Ad/(\pi n)} \]

Fig. 3. Relations between $d$ and $n$ for different levels of connectivity.

8/13/2009
Observations

- If we tolerate a small percent of nodes being isolated, the critical communication radius will be considerably reduced.
- This could result in reducing communication energy consumption significantly since energy $\propto \{\text{communication radius}^{q=2-4}\}$.
Outlines

- Introduction
- Asymptotic sink connectivity
- Critical communication radius for sink connectivity
- Effective communication radiiuses for different link models
Link models

- **Simple Boolean**
  \[(x_i, x_j) \text{ can communication with each other if and only if } \|x_i - x_j\| < r\text{, where } r \text{ is a constant.}\]

- **Random connection**
  \[x_i \text{ can send a message to } x_j \text{ with the probability } g(\|x_i - x_j\|)\]

- **Anisotropic**
  \[x_i \text{ can send a message to } x_j \text{ if and only if } \|x_i - x_j\| < r(\phi, \theta_i)\text{, see the figure.}\]

- **Random radius**
  \[(x_i, x_j) \text{ can communication with each other if and only if } \|x_i - x_j\| < r_i\text{, where } r_i \text{ is a random variable.}\]
Effective Communication Radius

\[ r_e = E(\sqrt{e(g)/\pi}) = E(\sqrt{\int_{x \in \mathbb{R}^2} g(x)dx / \pi}). \]

as the effective communication radius where \((x_i, x_j)\) is connected with probability \(g(x_i - x_j)\), \(e(g)\) is the effective communication area

Numerous of simulation results show that

- If the effective communication radius \(> R\), the sink connectivity of three other link models (or the combination of three other link models) is better than that of the simple Boolean model
- Note: Here the sink connectivity is the fraction of nodes that can receive the broadcasting messages from the sink.
Average connectivity for different link models

Fig. 4. relations between $C_n$ and $d$ in four different models.
Summary and conclusions

- Sink connectivity is proposed for wireless sensor networks
- If we tolerate a small fraction of nodes being isolated, we can reduce the communication radius, and thus the communication power consumption significantly.
- If the density of the nodes remain unchanged, the critical communication radius for sink connectivity would decrease opposite to the critical communication radius for full connectivity.
- Effective communication radius is introduced to describe the sink connectivity in more complicated link models.
Thank you