Anti-Jamming Schedules for Wireless Broadcast Systems

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Abstract

Modern society is heavily dependent on wireless networks for providing voice and data communications. Wireless data broadcast has recently emerged as an attractive way to disseminate data to a large number of clients. In data broadcast systems, the server proactively transmits the information on a downlink channel; the clients access the data by listening to the channel. Wireless data broadcast systems can serve a large number of heterogeneous clients, minimizing power consumption as well as protecting the privacy of the clients' locations.

The availability and relatively low cost of antennas resulted in a number of potential threats to the integrity of the wireless infrastructure. The existing solutions and schedules for wireless data broadcast are vulnerable to *jamming*, i.e., the use of active signals to prevent data distribution. The goal of jammers is to disrupt the normal operation of the broadcast system, which results in high waiting time and excessive power consumption for the clients.

In this paper we investigate efficient schedules for wireless data broadcast that perform well in the presence of a jammer. We show that the waiting time of client can be efficiently reduced by adding redundancy to the schedule. The main challenge in the design of redundant broadcast schedules is to ensure that the transmitted information is always up-to-date. Accordingly, we present schedules that guarantee low waiting time and low staleness of data in the presence of a jammer. We prove that our schedules are optimal if the jamming signal has certain energy limitations.

I. Introduction

Modern society has become heavily dependent on wireless networks to deliver information to diverse users. People expect to be able to access the latest data, such as stock quotes and traffic conditions, at any time, whether they are at home, at their office, or traveling. The emerging wireless infrastructure provides opportunities for new applications such as on-line banking and electronic commerce. Wireless data distribution systems also have a broad range of applications in military networks, such as transmitting up-to-date battle information to tactical commanders in the field. New applications place high demands on the quality, reliability, and security of transmissions. In order to provide a ubiquitous and powerful communication infrastructure that can satisfy security and reliability demands, sophisticated network technology, protocols and algorithms are required.

Due to their open and ubiquitous nature, wireless information systems are extremely vulnerable to attack and misuse. Wireless systems can be attacked in various ways, depending on the objectives and capabilities of an adversary. Due to high availability and relatively low cost of powerful antennas, *jamming*, i.e., the use of active signals to prevent data distribution, has emerged as an attractive way of attack. As the current data communication standards such as IEEE802.11 [1] and Bluetooth [2] are not designed to resist malicious interference, a small number of jammers with limited energy resources can disrupt operation of an entire network [21]. Jamming is a common method of attack in military networks, where transmissions are often performed in the presence of an adversary whose goal is to disrupt the communication to a maximum degree. For example, the Global Positioning System (GPS) relies on extremely weak signals from orbiting satellites and, as a result, is very vulnerable to jamming. This constitutes a significant threat for GPS-based weapon and navigational systems. Jamming can be viewed as a form of *Denial-of-Service* (DoS) attack, whose goal is to prevent users from receiving timely and adequate information.

A. Wireless Data Broadcast Systems

One common characteristic of wireless infrastructure is an asymmetry between the downlink and uplink channels. In cellular, 802.11, or others similar networks, the downlink channel is of much higher bandwidth than the uplink

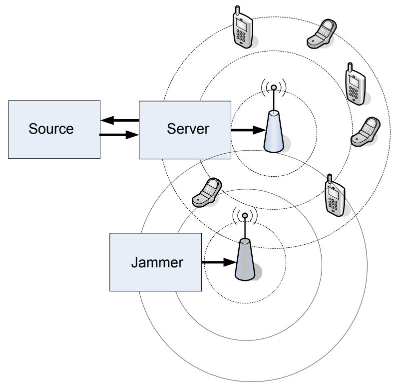

Fig. 1. A typical data broadcast system.

channel. Moreover, while the downlink channel is operated by a powerful antenna, the uplink channel is driven by a mobile device with limited power resources.

This intrinsic asymmetry of wireless systems impacts the way information is delivered to clients. In particular, the standard *client-server* paradigm, in which the data transfer is initiated by clients, is not adequate for wireless systems [3]. Wireless data broadcast [3], [6], [17] has recently emerged as an attractive way to disseminate data to a large number of clients. In data broadcast systems, the server proactively transmits the information on the downlink channel and the clients access data by listening to the channel. This approach enables the system to serve a large number of heterogeneous clients, minimizing client power consumption as well as protecting the privacy of the clients' locations.

Fig. 1 depicts a typical data broadcast system. The system includes the following components: the server (scheduler), the broadcast channel, the information source, and the wireless users. The server periodically accesses the information source, retrieves the most recent data, encapsulates it into a packet and sends the packet (or encoding thereof) over the broadcast channel.

There are two key performance characteristics of a wireless data distribution system. The first characteristic is *waiting time*, i.e., the amount of time spent by the client waiting for data. Waiting time is an important parameter, as timely information delivery is essential for many practical applications. In addition, it is closely related to the amount of power spent by the client to obtain the information. The second characteristic is *staleness*, i.e., the amount of time that passes from the moment the information is generated, until it is delivered to the client. The staleness of the schedule usually depends on the amount of redundancy used by the system, as information become less and less relevant with time.

B. Jamming Attacks

The goal of the jammer is to disrupt the normal operation of the broadcast system, which results in high waiting time and excessive power consumption of the clients. To that end, the jammer sends active signals over the channel that interfere with the signal sent by the server (see Fig. 1). The traditional defences against jamming include *spread spectrum* techniques such as *direct sequence* and *frequency hopping* [22], [24]. With direct sequence, the data signal is multiplied by a pseudo-random bit sequence, referred to as *pseudo-random noise code*. As a result, the signal is spread across a very wide bandwidth such that the amount of energy present at each particular frequency band is very small. In frequency hopping systems, the signal only occupies a single channel at any given point of time. The carrier frequency is constantly changing according to a unique sequence. Both techniques spread signal over a wide frequency band, which makes it harder for an adversary to find and jam the signal.

While spread-spectrum techniques constitute an important tool for combating jamming, an additional protection is required at packet-level. First, the pseudo-random noise code or frequency hopping sequence may be known to the adversary, as in the case of the standard wireless protocols such as IEEE802.11 and Bluetooth. Second, even if no information about the spread-spectrum protocol is available to the adversary, it can still destroy a small number of bits in each transmitted packet by sending a strong jamming signal of short duration. If no other protection mechanism is used at the packet-level, as in the case of IEEE802.11 and Bluetooth, the few destroyed bits will result in dropping of the entire packet. Accordingly, there is a need to provide an additional packet-level protection, which has to be built on top of traditional anti-jamming techniques.

Accordingly, in this paper we investigate efficient anti-jamming schedules for data broadcast. In our schedules, each packet is encoded by an error-correcting code, such as Reed-Solomon, which allows the schedule to minimize both waiting time of the clients and the staleness of the received data. As power supply is the most important constraint for practical jammers, we focus on jammers that have certain restrictions on the length of jamming pulses and the length of the intervals between subsequent jamming pulses. To the best of our knowledge, this is the first study that investigates anti-jamming schedules for wireless data distribution systems.

C. Related Work

The design of optimal broadcast schedules attracted a large body of research over past years. Ammar and Wong [4], [5] studied broadcast schedules for teletext systems. Vaidya and Hameed [14], [15], [23] established optimal broadcast schedules for sending packets generated by multiple information sources.

Scheduling of broadcast channels was studied in the sequence of works [8]–[12]. It was shown [8] that time division method, where packets are sent sequentially on a single full-bandwidth channel, performs better than frequency division method, where each information source has its own subchannel of lower bandwidth. Splitting packets into smaller pieces was investigated in [10]. Data broadcast over lossy communication channels was studied in [11]. This work proposes efficient coding solutions that reduce performance degradation due to packet loss. In [12] we study efficient broadcast schedules for multiple broadcast channels.

Studies [19], [20] focused on the design of *universal schedules* that guarantee low *waiting time* for any user, regardless of the access pattern. We show that a good universal schedule has to combine both encoding and randomization techniques. We show how to incorporate randomness and redundancy into the schedule and provide a way to identify an optimal schedule that satisfies given staleness requirements. In particular, we investigate the trade-off between the staleness and the waiting time and present schedules that yield the lowest possible waiting time for any given staleness constraint.

A more traditional *on-demand* model for data broadcast is a well-studied topic in theoretical computer science (see e.g.., [7], [13], [18] and references therein). In this model, clients request data on the uplink channel and the server responds by sending this data to the client on the downlink channel.

The rest of the paper is organized as follows. In Section II, we formally define the communication model in the presence of a jammer and state our results. In Section III, we consider an important class of regular schedules, in which the length of each message includes the same number of symbols. Next, in Section IV we establish upper and lower bounds on the performance of general (un-restricted) schedules. Finally, in Section V, we conclude with a few remarks and open problems.

II. Model

A. Schedules

As mentioned in the introduction, the data is delivered in the form of packets, each packet captures the current state of the information source. We assume that each packet includes exactly k information symbols. We also assume that transmission of k symbols of over the channel requires one unit of time.

We enumerate the packets, according to the time of their transmission. Each packet is encoded into a message that contains at least k symbols by using an a Maximal Distance Separable (MDS) code, such as a Reed-Solomon [16]. The encoding ensures that any k symbols of the message are sufficient in order to reconstruct the original message.

Definition 1 (Schedule $\mathcal{S}$): A *schedule* is a sequence $\{r_1, r_2, \dots\}$, $r_i \geq 1$, such that r_i is the amount of time required to transmit message i.

Note that the length of message i is equal to $r_i k$.

A schedule $\mathcal{S} = \{r_1, r_2, \dots\}$ can also be defined by its *transmission sequence* $\{t_1, t_2, \dots\}$, where t_i represents the starting time of the transmission of message i, i.e., $t_1 = 0$ and $t_i = \sum_{j=1}^{i-1} r_j$ for $i > 1$.

Example 2: Consider the schedules depicted in Fig. 2 (a) and (b). In the first schedule, each encoded message contains rk symbols. Thus, the schedule transmits each message is transmitted over an interval of r time units and generates a new packet at times $0, r, 2r, \dots$. The second schedule transmit messages of different length.

A wireless client begins to listen to the wireless channel upon a request for new information. In order to satisfy the request, the client must receive at least k symbols from the currently transmitted message. If the client fails to

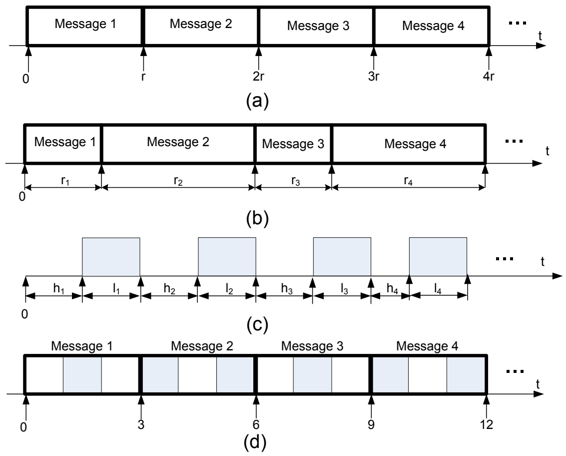

Fig. 2. Examples of schedules and jamming messages

receive k symbols from the current message, it continue to listen to the channel, until it receives at least k symbols from one of the subsequent messages.

The are two key performance characteristics of the schedule: *the expected waiting time* and the maximum staleness of the received data.

Definition 3 (Waiting time $WT_t(\mathcal{S})$): Let $\mathcal{S}$ be a broadcast schedule. Suppose that the client's request was placed at time t. Let n be the number of the message currently transmitted over the channel. Let t' be the first time the client receives at least k symbols from a message n', $n' \geq n$. Then, the waiting time of the client is defined as $WT_t = t' - t$.

Following [15], [12], and [18], we assume that the clients' requests are distributed uniformly over time. Accordingly, the expected waiting time of the clients is defined as follows:

Definition 4 (Expected Waiting Time $EWT(\mathcal{S})$): Let $\mathcal{S}$ be a broadcast schedule. Then, the *expected waiting time* is defined as follows:

$$EWT(\mathcal{S}) = \lim_{T\to\infty} \frac{1}{T}\int_0^T WT_t(\mathcal{S})dt \quad (1)$$

The waiting time is an extremely important parameter for many time-sensitive applications. In addition, it is closely related to the amount of power spent by the client to obtain the information.

The *staleness* of the data is defined to be the amount of time that passes from the moment the information is generated until it is delivered to the client. The staleness captures the quality of delivered information, because in dynamic settings the information becomes less and less relevant with time.

Definition 5 (Staleness $ST_t(\mathcal{S})$): Let $\mathcal{S}$ be a broadcast schedule. Suppose that the client's request was placed at time t. Let n be the number of the message currently transmitted over the channel. Further, let $n' \geq n$ be the first message for which the client receives at least k symbols. Then, the staleness of the data is defined to be $ST_t = t_{n'} - t$.

Example 6: Consider the schedule depicted in Fig. 2(a). Suppose that a client arrives at time t. The number of symbols received by the client from the currently transmitted message is equal to $n_t = (\lceil \frac{t}{r} \rceil r - t)k$. If $n_t \geq k$, then the client will be able to decode this message, hence its waiting time is zero. Otherwise, the client needs to wait for the next message, hence its waiting time is n_t. It is easy to verify that if the clients are distributed uniformly over time, the expected waiting time is $\frac{k}{2n} = \frac{1}{2r}$.

While redundant transmission improves the expected waiting time of a schedule, it comes at a price in terms of the staleness of the received data. Indeed, if $n_t \geq k$, then the packet received by the client at time t, was generated in time $\lfloor \frac{t}{r} \rfloor r$, hence the staleness of the data is $t - \lfloor \frac{t}{r} \rfloor r$. On the other hand, if $n_t < k$, then the client will get a new packet, hence the staleness is zero.

The example demonstrates that there exists a certain trade-off between waiting time and staleness in data broadcast systems. While finding a schedule that has minimum waiting time subject to a staleness constraint in a not-jammed channel is a relatively easy task, this task is much more complicated in the presence of a jammer.

Jammer model

The jamming model must be accurate enough to capture the characteristics of practical jammers, and, at the same time, be simple enough for the optimization of network protocols. In this paper we focus on a *pulse erasure jammer*. Such a jammer produces a sequence of pulses, each pulse results in an erasure in the channel.

Definition 7 (Jamming Sequence $\mathcal{J}$): A *jamming sequence* is a sequence $\{h_1, l_1, h_2, l_2, \dots\}$, such that h_1 is the beginning time of the first pulse, l_i is the length of pulse i, and h_i, $i \geq 2$ is the length of time interval between pulses $i-1$ and i.

Fig. 2(c) depicts an example of a jamming sequence.

It has been recognized [21] that the power supply is the most important limitation for the majority of practical jammers. A typical jammer is powered by a battery, which can be recharged from an external source, such as a solar cell array. Accordingly, in our model, we limit the length of pulses in the jamming sequence by a constraint $l^{\max}$, i.e., $l_i \leq l^{\max}$ for all $i \geq 1$. Since after each pulse the battery must be recharged we also constrain the length of the interval between two consecutive pulses to be at least $h^{\min}$, i.e., $h_i \geq h^{\min}$ for all $i \geq 2$.

We denote by $WT_t(\mathcal{S}, \mathcal{J})$ the waiting time of schedule $\mathcal{S}$ in the presence of jammer $\mathcal{J}$. Similarly, the expected waiting time of a schedule $\mathcal{S}$ in the presence of jammer $\mathcal{J}$ is denoted by $EWT(\mathcal{S}, \mathcal{J})$.

Example 8: Let $\mathcal{S}$ be a schedule $\{3, 3, \dots\}$ and $\mathcal{J}$ be a jamming sequence $\{1, 1, 1, \dots\}$ (see Fig. 2(d)). Then, the expected waiting time of a schedule $\mathcal{S}$ in the presence of jammer $\mathcal{J}$ is equal to $EWT(\mathcal{S}, \mathcal{J}) = 11/12$. Note that the expected waiting time of the schedule without the jammer is $\frac{1}{6}$. As we show later, $\mathcal{J}$ is not an optimal jamming sequence for $\mathcal{S}_3$ - the most efficient jammer can achieve the waiting time of $\frac{23}{18}$.

In this paper we focus on jamming sequences with power limitations $h^{\min} = l^{\max} = 1$. In this case the length of jamming pulses is comparable with the time required for transmitting a single packet. Beside being an interesting case *per se*, the techniques and the tools we develop can be easily extended to more general cases. We refer to a jammer (jamming sequence) that satisfies the energy limitations as an *admissible* jammer (jamming sequence).

Results

We focused on finding optimal jamming sequences for the broad class of *regular* schedules $\{\mathcal{S}_r \mid r \geq 1\}$, where $\mathcal{S}_r = \{r, r, \dots\}$. A regular schedule $\mathcal{S}_r$ transmits each message over a time interval of length r time units. Thus, each message in schedule $\mathcal{S}_r$ contains rk symbols. The advantage of regular schedules is that they provide firm guarantees on the staleness of the received data. Specifically, schedule $\mathcal{S}_r$ ensures that the staleness of the received information is at most $r-1$ time units. In addition, regular schedules are easier to implement than a broader class of schedules in which the length of each message can vary. With regular schedules we can use the same decoding algorithm, which simplifies the design of the mobile device and reduces its cost.

The optimal jamming sequence for a regular schedule $\mathcal{S}_r = \{r, r, \dots\}$ depends on the value of r. Specifically, for $r = 4$ the optimal jamming sequence is $\mathcal{J}_4 = \{1-\varepsilon, 1, 1, \dots\}$, where $\varepsilon = \frac{1}{k}$ (see Fig. 3(a)). This sequence is optimal for any even r. For $r = 5$, the optimal jamming sequence is $\mathcal{J}_5 = \{1-\varepsilon, 1, 1+2\varepsilon, 1, 1-2\varepsilon, 1, 1+2\varepsilon, 1, 1-2\varepsilon, \dots\}$, where $\varepsilon = \frac{1}{k}$ (see Fig. 3(b)). A similar type of jamming sequence is optimal for any odd $r \geq 5$. The schedule $\mathcal{S}_3$ requires a different type of the jamming sequence, as depicted on Fig. 3)(c).

If r is not an integer, the optimal jamming sequence $\mathcal{J}_r$ for $\mathcal{S}_r$ is typically obtained by modifying the optimal schedule for either $\lfloor r \rfloor$ or $\lceil r \rceil$. For example, if the integer part of r is an even number, then $\mathcal{J}_r$ is formed from $\mathcal{J}_{\lfloor r \rfloor}$ by increasing non-jamming intervals that include the boundary between two messages (see Fig. 3(d)).

The following theorem establishes upper bounds on the value of $EWT^{\max}(\mathcal{S}_r)$.

Theorem 9: Let $\mathcal{S}_r$ be a regular schedule.

1) If $2 \leq r < 3$, then the maximal waiting time achievable by an admissible jammer is at most

$$EWT^{\max}(\mathcal{S}_r) \leq 1 + \frac{2}{r}. \tag{2}$$

2) If $r \geq 4$ and the integer part $\lfloor r \rfloor$ of r is an even number, then the maximal waiting time achievable by an admissible jammer is at most

$$EWT^{\max}(\mathcal{S}_r) \leq \frac{3}{4} + \frac{r - \lfloor r \rfloor + 10}{4r}. \tag{3}$$

3) If $r \geq 3$ and the integer part of r is an odd number, the worst case waiting time $EWT^{\max}(\mathcal{S}_r)$ achievable by an admissible jammer is at most $EWT^{\max}(\mathcal{S}_r) \leq \frac{3}{4} + \frac{10+2\delta^2-\delta}{4r}$, where $\delta = 2\lceil \frac{r}{2} \rceil - r$.

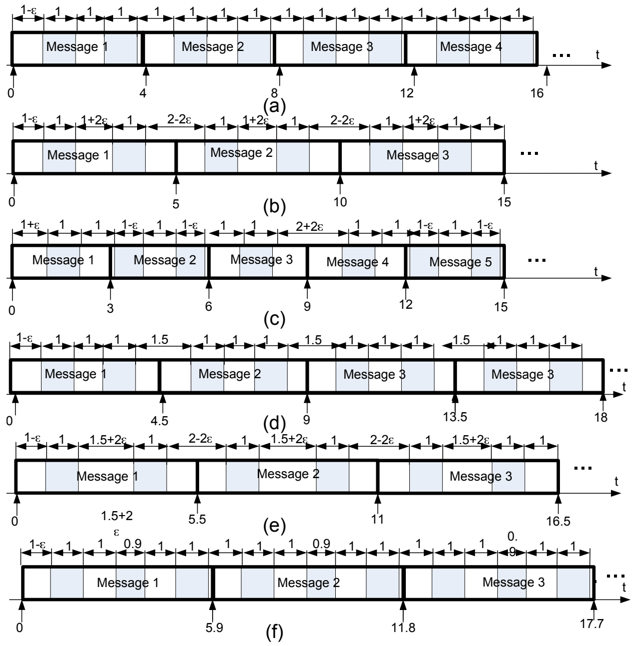

Fig. 3. Jamming sequences for different values of r

Proof: See Section III-C. ■

In the next theorem we establish lower bounds on the value of $EWT^{\max}(\mathcal{S}_r)$.

Theorem 10: Let $\mathcal{S}_r$ be a regular schedule. Then, up to terms of order $\varepsilon = \frac{1}{k}$,

$$EWT^{\max}(\mathcal{S}_r) \geq \begin{cases} 1 + \frac{2}{r} & \textbf{if } 2 \leq r < 3 \\ \frac{23}{6r} & \textbf{if } 3 \leq r < 1 + \sqrt{\frac{51}{3}} \\ \frac{r^2-2r+3}{2r} & \textbf{if } 1 + \sqrt{\frac{51}{3}} \leq r < 4 \\ \frac{3}{4r} + \frac{10+r-\lfloor r \rfloor}{4r} & \textbf{if } 2i \leq r < 2i+1, \\ & \quad i = 2, 3, \ldots \\ \frac{3\lfloor r \rfloor + 11}{4r} & \textbf{if } 2i+1 \leq r < 2i + \sqrt{3}, \\ & \quad i = 2, 3, \ldots \\ \frac{3}{4} + \frac{2\delta^2 - 5\delta + 10}{4r} & \textbf{if } 2i + \sqrt{3} \leq r < 2(i+1), \\ & \quad i = 2, 3, \ldots \end{cases} \quad (4)$$

where $\delta = \lceil r \rceil - r$.

Proof: See Section III-D. ■

We prove Theorem 10 by constructing jamming sequences that yield the desirable values of the expected waiting times. For values of r that satisfy $2i+1 \leq r \leq 2i+\sqrt{3}$, $i = 2, 3, \ldots$, such sequences are formed from the optimal schedules for $\lfloor r \rfloor$ by increasing one of the non-jamming intervals in each message, as depicted in Fig. 3(e). For values of r that satisfy $2i + \sqrt{3} \leq r \leq 2(i+1)$, $i = 2, 3, \ldots$, we add a jamming pulse in the middle of each message, as depicted in Fig. 3(f).

Table I summarizes the lower and upper bounds on $EWT^{\max}(\mathcal{S}_r)$ for a broad range of values of r. The lower and upper bounds on $EWT^{\max}(\mathcal{S}_r)$ are also depicted in Fig. 4. The proof of the upper bound for the special case of $r = 3$ is rather involved and omitted from this version due to the space constraints.

It is important to note that the size of the message r in a regular schedule $\mathcal{J}_r$ is closely related to the staleness of the delivered information. Indeed, the maximum staleness of the data is always lower or equal to $r-1$, while its average staleness does not exceed $\frac{r-1}{r}$. Hence our results establish a trade-off between the expected waiting time of the clients and the staleness. In particular, we identify the best schedule for any given staleness constraint.

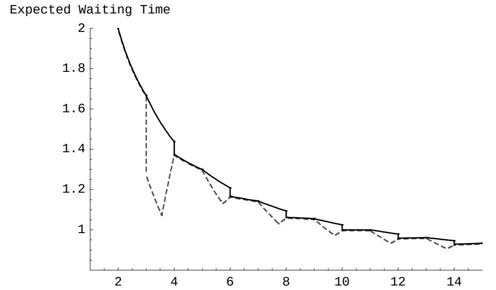

Fig. 4. The lower and upper bounds on $EWT^{\max}(\mathcal{S}_r)$. The lower and upper bounds are marked by solid and dashed lines, respectively.

We observe that the schedule $\mathcal{J}_3$ has a clear advantage over other schedules: it achieves low expected waiting time with minimum penalty in terms of the staleness of the delivered data.

Schedule	Lower Bound	Upper Bound
$r < 2$	∞	∞
$2 \le r < 3$	$1 + \frac{2}{r}$	$1 + \frac{2}{r}$
$r = 3$	$\frac{23}{18}$	$\frac{23}{18}$
$3 < r < 1 + \frac{\sqrt{51}}{3}$	$\frac{23}{6r}$	$\frac{3}{4} + \frac{2\delta^2 - \delta + 10}{4r}$
$1 + \frac{\sqrt{51}}{3} \le r < 4$	$\frac{r^2 - 2r + 3}{2r}$	$\frac{3}{4} + \frac{2\delta^2 - \delta + 10}{4r}$
$2i \le r < 2i + 1$, $i = 2, 3 \ldots$	$\frac{3}{4} + \frac{10 + r - \lfloor r \rfloor}{4r}$	$\frac{3}{4} + \frac{10 + r - \lfloor r \rfloor}{4r}$
$2i + 1 \le r < 2i + \sqrt{3}$ $i = 2, 3, \ldots$	$\frac{3\lfloor r \rfloor + 11}{4r}$	$\frac{3}{4} + \frac{2\delta^2 - \delta + 10}{4r}$
$2i + \sqrt{3} \le r < 2(i+1)$ $i = 2, 3, \ldots$	$\frac{3}{4} + \frac{2\delta^2 - 5\delta + 10}{4r}$	$\frac{3}{4} + \frac{2\delta^2 - \delta + 10}{4r}$

TABLE I

THE PERFORMANCE CHARACTERISTICS OF OPTIMAL SCHEDULES. HERE, $\delta = 2\lceil \frac{r}{2} \rceil - r$

In addition, we established upper and lower bounds on the worst case waiting time $EWT^{\max}(\mathcal{S})$ for a general class of non-regular schedules. This class includes schedules in which the length of each message is different and the schedules that employ randomization, i.e., the length of each message is distributed according to some probability distribution. We assume that in the case of random schedules the jammer knows the probability distribution but has no access to the server's random bits.

Theorem 11: Let $\mathcal{S}$ be a schedule and let r be the expected length of the messages in $\mathcal{S}$. Then the worst case expected waiting time $EWT^{\max}(\mathcal{S})$ of the schedule in the presence of an admissible jammer is bounded by

$$\frac{3}{4} + \frac{3}{2r} \le EWT^{\max}(\mathcal{S}) \le \frac{3}{4} + \frac{11}{4r} \tag{5}$$

III. REGULAR ANTI-JAMMING SCHEDULES

In this section we analyze regular schedules $\mathcal{S}_r = \{r, r, \ldots\}$. In particular, we establish lower and upper bounds on $EWT^{\max}(\mathcal{S}_r)$ for all $r > 0$.

A. Definitions

We begin by introducing several definitions. Let $\mathcal{S}$ be a schedule and $\mathcal{J}$ be an admissible jamming sequence. We refer to the beginning of the transmission of a new message as an *update* point. Next, the end point of each jamming pulse is referred to as a *jamming* point.

Definition 12 (Block): Given a schedule $\mathcal{S}$ and a jamming sequence $\mathcal{J}$, a *block* is a time interval $[t', t'']$, such that t' is either an update or a jamming point and t'' is the next closest update or jamming point.

Definition 13 (Waiting Times for an Interval): Let $\mathcal{S}$ be a schedule, $\mathcal{J}$ be an admissible jamming sequence, and $I = [t', t'']$ be a time interval. Then, we define the following waiting times for $\mathcal{S}$ and $\mathcal{J}$ on the interval I:

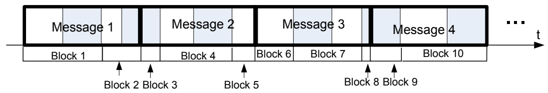

Fig. 5. Blocks in the schedule.

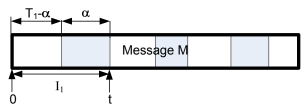

Fig. 6. The first block in the message.

1) Waiting time $WT(I)$ on the interval I, $WT(I)(\mathcal{S}) = \int_{t'}^{t''} WT_t(\mathcal{S})dt$;
2) Waiting time with jamming $JWT(I)$ on the interval I, $JWT(I)(\mathcal{S}, \mathcal{J}) = \int_{t'}^{t''} WT_t(\mathcal{S}, \mathcal{J})dt$;
3) Added waiting time. The added waiting time AWT_t is defined to be

$$AWT_t = WT_t(\mathcal{S}, \mathcal{J}) - WT_t(\mathcal{S}). \tag{6}$$

For an interval I we define $AWT(I) = WTJ(I) - WT(I)$;
4) Average Additional waiting time $AAWT(I)$ for schedule $\mathcal{S}$ in the presence of jamming sequence $\mathcal{J}$ on the interval I, $AAWT(I) = \frac{AWT(I)}{t''-t'}$.

B. *Added waiting time for a single message*

In the following lemmas we are analyzing the added waiting time of the clients that arrive during the transmission of message i. Let I be the time interval allocated for message i by the schedule. We divide I into four subintervals I_1, I_2, I_3, and I_4, such that I_1 includes the first block of I, I_2 includes all blocks of I except the first block and the last two blocks, and I_3 and I_4 include the last two blocks of I.

We begin by establishing an upper bound on the added waiting time for I_1.

Lemma 14: Let T_1 be the length of interval I_1. Suppose that there is no update point in the time unit that follows I_1. Then the added waiting time $AWT(I_1)$ of I_1 is bounded by

$$AWT(I) \le \begin{cases} \frac{T_1^2}{2} & \textbf{if } T_1 \le 1 \\ T_1 - \frac{1}{2} & \textbf{if } 1 \le T_1 \le 2 \\ \frac{3}{2} & \textbf{if } T_1 \ge 2 \end{cases}$$

In particular, $AAWT(I_1) \le \frac{3}{4}$.

Proof: Let α be the length of the jammed portion of the block as in Fig. 6. Note that $0 \le \alpha \le \min\{T_1, 1\}$. If $T_1 \le 2$, we have:

$$AWT(I_1) = \begin{cases} (T_1 - \alpha)\alpha + \frac{\alpha^2}{2} & \textbf{if } T_1 \le 1 + \alpha \\ \alpha + \frac{\alpha^2}{2} & \textbf{if } T_1 \ge 1 + \alpha \end{cases}$$

Since $0 \le \alpha \le 1$, and $T_1 \ge 1$, the value of $AWT(I_1)$ is maximized for

$$\alpha = \begin{cases} T_1 & \textbf{if } T_1 \le 1 \\ 1 & \textbf{if } T_1 \ge 1 \end{cases} \tag{7}$$

Hence,

$$AWT(I_1) \le \begin{cases} \frac{T_1^2}{2} & \textbf{if } T_1 \le 1 \\ T_1 - \frac{1}{2} & \textbf{if } 1 \le T_1 \le 2 \\ \frac{3}{2} & \textbf{if } T_1 \ge 2 \end{cases}$$

It is easy to verify that the maximum value of $AAWT(I_1)$ is $\frac{3}{4}$. ■

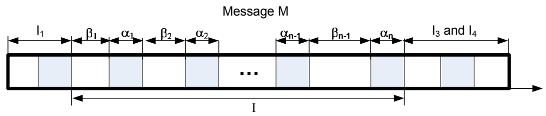

Fig. 7. The second subinterval.

Lemma 15: Let T_2 be the length of interval I_2. Then,

$$AWT(I_2) \leq \begin{cases} \frac{3T_2+2\delta^2-5\delta}{4} & \textbf{if } \lfloor T_2 \rfloor \text{ is odd} \\ \frac{3}{4}(\lfloor T_2 \rfloor) & \textbf{otherwise} \end{cases}$$

where $\delta = \lceil T_2 \rceil - T_2$. In particular, $AWT(I_2) \leq \frac{3}{4}T_2$, and therefore $AAWT(I_2) \leq \frac{3}{4}$.

Proof: We denote by $I_2^1, \ldots, I_2^n$ the blocks included in the interval I_2. As shown in Fig. 7, let β_i be the length of the unjammed part of block I_2^i, and α_i be the length of the jammed part of that block. Note that for all i it holds that $\beta_i \geq 1$, and $\alpha_i \leq 1$. Hence,

$$AWT(I_2^i) = \alpha_i + \frac{\alpha_i^2}{2},$$

which, in turn, implies that

$$AWT(I_2) = \sum_{i=1}^{n} \alpha_i + \frac{1}{2}\sum_{i=1}^{n} \alpha_i^2.$$

Note that $AWT(I_2)$ does not depend on β_i, hence we can assume without loss of generality that $\beta_i = 1$ for $i \neq 1$. If there exists $i > 1$ such that $\beta_i > 1$, we remove an unjammed interval of length $\beta_i - 1$ from block I_2^i, and add it at the beginning of the interval I_2. Note that this change in the jamming sequence does not decrease the value of $AWT(I_2)$. This is due to the fact that the added waiting time of the initial subinterval $I_2'^i$ of I_2^i of length $\beta_i - 1$ is zero.

We conclude that

$$T_2 = \sum_{i=1}^{n} \alpha_i + \sum_{i=2}^{n} 1 + \beta_1,$$

which implies that

$$\sum_{i=1}^{n} \alpha_i = T_2 - n + 1 - \beta_1,$$

or, equivalently,

$$AWT(I_2) = T_2 - n + 1 - \beta_1 + \frac{1}{2}\sum_{i=1}^{n} \alpha_i^2.$$

First let us consider the case in which $\lfloor T_2 \rfloor$ is an odd number. Since $\alpha_i \leq 1$ for all i, the value of $AWT(I_2)$ is maximized when $\alpha_i = 1$ for $i < n$, $\alpha_n = 1 - \delta$, $\beta_1 = 1$, and $n = k$. Hence, the maximum added waiting time is

$$AWT(I_2) = T_2 - n + \frac{1}{2}\sum_{i=1}^{n} \alpha_i^2 =$$

$$= 2n - 2 - \delta - n + \frac{1}{2}(n - 1 + (1-\delta)^2) =$$

$$= \frac{6n - 3\delta + (2\delta^2 - 5\delta)}{4} = \frac{3T_2 + 2\delta^2 - 5\delta}{4}.$$

We note that $2\delta^2 - 5\delta \leq 0$, hence $AWT(I_2) \leq \frac{3}{4}T_2$.

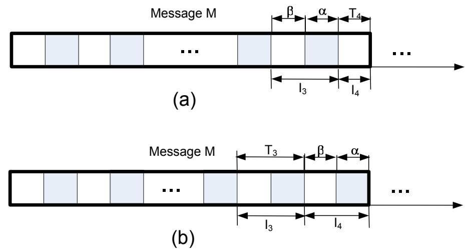

Fig. 8. The last two blocks.

Next we consider the case in which $\lfloor T_2 \rfloor$ is an even number. In this case the value of $AWT(I_2)$ is maximized when $\alpha_i = 1$ for $i < n$, $\alpha_n = 1$, $\beta_1 = 1 + \delta$, and $n = k$. Thus,

$$AWT(I_2) = T_2 - n - \delta + \frac{1}{2}\sum_{i=1}^{n} \alpha_i^2 =$$

$$= 2n + \delta - n - \delta + \frac{1}{2}n$$

$$= \frac{6n + 3\delta - 3\delta}{4} = \frac{3}{4}(T_2 - \delta).$$

Since $\delta \geq 0$, we have $AWT(I_2) \leq \frac{3}{4}T_2$. We conclude that $AAWT(I_2) = AWT(I_2)/|I_2| \leq \frac{3T_2}{4T_2} = \frac{3}{4}$. ■

Lemma 16: Let $T_4 = |I_4|$, $T_3 = |I_3|$. Then

$$AWT(I_3) \leq \begin{cases} \frac{2T_4(T_3-2)+T_3^2+2T_3-1}{2} & \textbf{if } T_4 < 1 \text{ and } T_3 \leq 2 \\ 7/2 & \textbf{if } T_4 < 1 \text{ and } T_3 \geq 2 \\ \frac{T_3^2-1}{2} & \textbf{if } T_4 \geq 1 \text{ and } T_3 \leq 2 \\ 3/2 & \textbf{if } T_4 \geq 1 \text{ and } T_3 \geq 2 \end{cases}$$

$$AWT(I_4) \leq \begin{cases} T_4 & \textbf{if } T_4 < 1 \\ \frac{-T_4^2+4T_4-1}{2} & \textbf{if } 1 \leq T_4 \leq 2 \\ 3/2 & \textbf{if } T_4 \geq 2 \end{cases}$$

In particular, if $T_4 < 1$, then $AAWT(I_3) \leq 7/4$, and $AAWT(I_4) \leq 1$. If $T_4 \geq 1$, then $AAWT(I_3) \leq 3/2$, $AWT(I_3) \leq 3/4$, $AWT(I_4) \leq 3/2$, $AWT(I_4) \leq 3/2$, and $AAWT(I_4) \leq 1$.

Proof: Let us assume that there is a jamming pulse of length one at the end of I_4, located less than one time unit away from the update point. We can make this assumption since it does not decrease the values of $AWT(I_4)$ and $AWT(I_3)$.

We begin by considering the case in which $T_4 < 1$. In this case interval I_4 must be unjammed, and the AWT_t for any $t \in I_4$ is at most 1. Therefore, $AWT(I_4) \leq T_4$, which, in turn, implies that $AAWT(I_4) = AWT(I_4)/T_4 \leq 1$. As in Fig. 8a, let α and β be the lengths of the jammed and unjammed portions of I_3 respectively. Assume first that $T_3 \leq 2$. Note that $\beta \geq 1$, and if $\beta > 1$, then the $AAWT(I_4)$ would be worse since we would have the same $AWT(I_4)$, but smaller length. Recall that $\alpha + \beta = T_3$, we implies that $\alpha = T_3 - 1$. We conclude that

$$AWT(I_3) = \alpha T_4 + \alpha(2 + \frac{\alpha}{2}) + 1 - T_4 =$$

$$= T_4(\alpha - 1) + \frac{\alpha^2}{2} + 2\alpha + 1 = \frac{2T_4(T_3 - 2) + T_3^2 + 2T_3 - 1}{2}.$$

Since $T_3 \leq 2$, we have $AWT(I_3) \leq \frac{T_3^2+2T_3-1}{2}$. The value of $AWT(I_3)$ is maximized when $T_3 = 2$, since it $AWT(I_3)$ is an increasing function of T_3. This implies that $AWT(I_3) \leq 7/2$. If $T_3 > 2$, then AWT_t for any t that belong to the first $T_3 - 2$ units of T_3 is zero, hence it still holds that $AWT(I_3) \leq 7/2$.

The $AAWT(I_3)$ is:

$$AAWT(I_3) \leq \frac{T_3^2 + 2T_3 - 1}{2T_3}.$$

We want to maximize this for $1 \leq T_3 \leq 2$. Taking the derivative with respect to T_3, we have

$$\frac{\delta}{\delta T_3} AAWT(I_3) = \frac{T_3^2 + 1}{2T_3^2}.$$

So the derivative is always positive for $1 \leq T_3 \leq 2$, so the maximum over this interval is achieved for $T_3 = 2$. Therefore if $T_4 < 1$, $AAWT(I_3) \leq 7/4$.

Now consider the case $T_4 \geq 1$. In this case I_4 begins with an unjammed interval of length at least one time unit which does not contain an update point. Thus, I_3 is in the same situation as the "middle" blocks. Let $T_3 = 2 - \delta$, then $AWT(I_3) \leq \frac{3(2-\delta)+2\delta^2-5\delta}{4} = \frac{T_3^2-1}{2}$, and $AAWT(I_3) \leq 3/4$. Now look at I_4, again, as in Fig. 8b, let α and β be the lengths of the jammed and unjammed portions of I_3 respectively, and note that we may assume, without loss of generality, that $\beta = 1$. Moreover, the best we can hope for is that the $1 - \alpha$ time units following the update at the end of I_4 will be jammed. In this case, we have:

$$AWT(I_4) = \alpha \left(2 - \frac{\alpha^2}{2}\right) + 1 - \alpha = \frac{2 + 2\alpha - \alpha^2}{2}.$$

But $T_4 = \alpha + 1$, so substituting, we get:

$$AWT(I_4) = \frac{-T_4^2 + 4T_4 - 1}{2}.$$

For $1 \leq T_4 \leq 2$, the $AWT(I_4)$ is maximized for $T_4 = 2$, which gives $AWT(I_4) \leq 3/2$. If $T_4 > 2$, then the AWT for the first $T_4 - 2$ time units will be zero, therefore $AWT(I_3) \leq 3/2$.

Now let us look at the $AAWT(I_4)$:

$$AAWT(I_4) = \frac{-T_4^2 + 4T_4 - 1}{2T_4}.$$

Taking the derivative of the $AAWT(I_4)$ with respect to T_4, we have:

$$\frac{\delta}{\delta T_4} AAWT(I_4) = \frac{1 - T_4^2}{2T_4^2},$$

which is negative for $1 < T_4 \leq 2$. Since $AAWT(I_4)$ is maximized for $T_4 = 1$, we have $AAWT(I_4) \leq 1$.

■

C. Upper bounds

In this section we establish upper bounds on the optimal jamming sequences for regular schedules. Recall that in a regular schedule $\mathcal{J}_r$ the length of each message is r time units.

We begin with the proof of Theorem 9.

Theorem 9: Let $\mathcal{S}_r$ be a regular schedule.

1) If $2 \leq r < 3$, then the maximal waiting time achievable by an admissible jammer is at most

$$EWT^{\max}(\mathcal{S}_r) \leq 1 + \frac{2}{r}. \quad (8)$$

2) If $r \geq 4$ and the integer part $\lfloor r \rfloor$ of r is an even number, then, the maximal waiting time achievable by an admissible jammer is at most

$$EWT^{\max}(\mathcal{S}_r) \leq \frac{3}{4} + \frac{r - \lfloor r \rfloor + 10}{4r}. \quad (9)$$

3) If $r \geq 3$ and the integer part of r is an odd number, the worst case waiting time $EWT^{\max}(\mathcal{S}_r)$ achievable by an admissible jammer is at most $EWT^{\max}(\mathcal{S}_r) \leq \frac{3}{4} + \frac{10+2\delta^2-\delta}{4r}$, where $\delta = 2\lceil \frac{r}{2} \rceil - r$.

Proof: Let $\mathcal{S}_r$ be a regular schedule and $\mathcal{J}$ be any admissible jamming sequence. Also, let M be any message of $\mathcal{S}$. Our goal is to establish an upper bound on the value of $AAWT(M)$.

We divide M into four intervals $I_1, \ldots, I_4$, such that I_1 contains the first block of M, I_2 contains all blocks of M except the first one and the last two; and I_3 and I_4 contain the last two blocks of M. We denote by $T_i = |I_i|$ for $i = 1, 2, 3, 4$. We can assume without loss of generality that intervals I_1, I_3, or I_4 do not contain an unjammed interval whose length is longer than one time unit. Indeed, if this is the case, such an interval can be shortened at the expense of one of the unjammed intervals in I_2, with no increase in the value of $AAWT(M)$. This implies that $T_1, T_3, T_4 \leq 2$. Then, by Lemmas 14, 15, and 16 it holds that

$$AWT(I_1) \leq \begin{cases} T_1^2/2 & \textbf{if } T_1 \leq 1 \\ \\ T_1 - 1/2 & \textbf{if } 1 \leq T_1 \leq 2 \end{cases} \tag{10}$$

$$AWT(I_2) \leq \begin{cases} \frac{3T_2 + 2\delta^2 - 5\delta}{4} & \textbf{if } \lfloor T_2 \rfloor \text{ is odd,} \\ \frac{3}{4}(\lfloor T_2 \rfloor) & \textbf{otherwise}, \end{cases} \tag{11}$$

where $\delta = \lceil T_2 \rceil - T_2$.

$$AWT(I_3) \leq \begin{cases} \frac{2T_4(T_3 - 2) + T_3^2 + 2T_3 - 1}{2} & \textbf{if } T_4 < 1 \\ \\ \frac{T_3^2 - 1}{2} & \textbf{if } T_4 \geq 1 \end{cases} \tag{12}$$

$$AWT(I_4) \leq \begin{cases} T_4 & \textbf{if } T_4 < 1 \\ \\ \frac{-T_4^2 + 4T_4 - 1}{4} & \textbf{if } T_4 \geq 1 \end{cases} \tag{13}$$

The added waiting time for message M equals to the sum of the added waiting times for its subintervals $I_1, \ldots, I_4$: $AWT(M) = AWT(I_1) + AWT(I_2) + AWT(I_3) + AWT(I_4)$.

An upper bound on $AWT(M)$ can be found by solving the following maximization program:

$$\begin{array}{lll} \text{maximize} & AWT(M) & \\ \text{subject to} & & \\ & T_1 + T_2 + T_3 + T_4 = r & \\ & T_i \geq 0 & i = 1, \ldots, 4 \\ & T_i \leq 2 & i = 1, 2, 4 \end{array} \tag{14}$$

It can be shown, using the tools of the theory of constrained optimization that for $r \geq 3$, the optimal value of $AWT(M)$ is achieved when the following conditions are satisfied:

$$\begin{array}{l} T_1 = r - T_2 - T_3 - T_4; \\ T_2 = 2\left\lfloor \frac{r-3}{2} \right\rfloor; \\ T_3 = 2; \\ T_4 = 1. \end{array} \tag{15}$$

If $r \geq 4$ and $\lfloor r \rfloor$ is an even number Equation 15 implies the following upper bound on $AWT(M)$:

$$AWT(M) \leq \frac{3r + 8 + r - \lfloor r \rfloor}{4},$$

which, in turn, implies that

$$AAWT(M) \leq \frac{3}{4} + \frac{r - \lfloor r \rfloor + 8}{4r}. \tag{16}$$

Since for regular schedules it holds that $EWT^{\max}(\mathcal{S}_r) \leq AAWT(M) + \frac{1}{2r}$, then we have

$$EWT^{\max}(\mathcal{S}_r) \leq \frac{3}{4} + \frac{r - \lfloor r \rfloor + 10}{4r}.$$

If $r \geq 3$ and $\lfloor r \rfloor$ is an odd number, Equation 15 yields the following upper bound on $AWT(M)$:

$$AWT(M) \leq \frac{3r + 8 - \delta + 2\delta^2}{4},$$

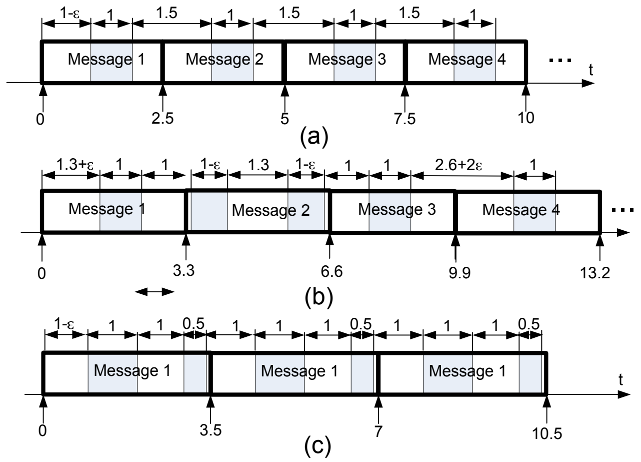

Fig. 9. Jamming sequences for regular schedules $\mathcal{J}_r$ (a) r=2.5 (b) r=3.3 (c) r=3.5

where $\delta = 2\lceil \frac{r}{2} \rceil - r$. This, in turn, implies that

$$AAWT(M) \leq \frac{3}{4} + \frac{2\delta^2 - \delta + 8}{4r}. \tag{17}$$

We conclude that

$$EWT^{\max}(\mathcal{S}_r) \leq \frac{3}{4} + \frac{2\delta^2 - \delta + 8}{4r},$$

where $\delta = 2\lceil \frac{r}{2} \rceil - r$.

If $2 \leq r < 3$, then solving optimization program (14) shows that $AAWT(M)$ is maximized for $T_3 = 2$, and $T_4 = r - 2$, which implies $T_1 = T_2 = 0$, since $r = T_1 + T_2 + T_3 + T_4$. So we have:

$$AAWT(M) \leq \frac{7 \cdot 2 + 4(r-2)}{4r} = \frac{3 + 2r}{2r} = 1 + \frac{3}{2r},$$

which implies

$$EWT^{max}(\mathcal{S}_r) \leq 1 + \frac{2}{r}.$$

■

D. Lower bounds

In this section, we prove Theorem 10.

Theorem 10: Let $\mathcal{S}_r$ be a regular schedule. Then, up to terms of order $\varepsilon = \frac{1}{k}$,

$$EWT^{\max}(\mathcal{S}_r) \geq \begin{cases} 1 + \frac{2}{r} & \textbf{if } 2 \leq r < 3 \\ \frac{23}{6r} & \textbf{if } 3 \leq r < 1 + \sqrt{\frac{51}{3}} \\ \frac{r^2 - 2r + 3}{2r} & \textbf{if } 1 + \sqrt{\frac{51}{3}} \leq r < 4 \\ \frac{3}{4r} + \frac{10 + r - \lfloor r \rfloor}{4r} & \textbf{if } 2i \leq r < 2i + 1, \\ & \quad i = 2, 3, \ldots \\ \frac{3\lfloor r \rfloor + 11}{4r} & \textbf{if } 2i + 1 \leq r < 2i + \sqrt{3}, \\ & \quad i = 2, 3, \ldots \\ \frac{3}{4} + \frac{2\delta^2 - 5\delta + 10}{4r} & \textbf{if } 2i + \sqrt{3} \leq r < 2(i+1), \\ & \quad i = 2, 3, \ldots \end{cases} \tag{18}$$

where $\delta = \lceil r \rceil - r$.

Proof: We prove the theorem by presenting, for each schedule $\mathcal{S}_r$, $r \geq 2$ a jamming sequence $\mathcal{J}_r$ such that $EWT(\mathcal{S}_r, \mathcal{J}_r)$ is equal to lower bound values stated in the theorem.

- For $2 \leq r < 3$, $\mathcal{J}_r = \{1-\varepsilon, 1, 1+\delta, 1, 1+\delta, \dots\}$, where $\delta = r - 3$ and $\varepsilon = \frac{1}{k}$. An example of this schedule for $r = 2.5$ is depicted on Fig. 9(a). Note that by Theorem 9 this schedule is optimal.
- For $3 \leq r < 1 + \frac{\sqrt{51}}{3}$, $\mathcal{J}_r = \{1+\delta+\varepsilon, 1, 1, 1-\varepsilon, 1+\delta, 1-\varepsilon, 1, 1, 2+2\varepsilon+2\delta, 1, 1, 1-\varepsilon, 1+\delta, 1-\varepsilon, 1, 1, 2+2\varepsilon+2\delta, \dots\}$, where $\delta = r - \lfloor r \rfloor$ and $\varepsilon = \frac{1}{k}$. This schedule is depicted on Fig. 9(b).
- For $1 + \frac{\sqrt{51}}{3} \leq r < 4$, $\mathcal{J}_r = \{1-\varepsilon, 1, 1, \delta, 1, 1, 1, \delta, \dots$, where $\varepsilon = \frac{1}{k}$ and $\delta = r - 3$.
- For $2i \leq r < 2i+1$, $i = 2, 3, \dots$, $\mathcal{J} = \{1-\varepsilon, 1, 1+2\varepsilon, 1 \cdots 1, 1+\delta-2\varepsilon, 1, 1+2\varepsilon, 1, \cdots, 1, 1+\delta-2\varepsilon, \cdots\}$, where $\varepsilon = \frac{1}{k}$ and $\delta = r - \lfloor r \rfloor$. An example of this schedule for $r = 4.5$ is depicted on Fig. 3(d).
- For $2i+1 \leq r < 2i+\sqrt{3}$, $i = 2, 3, \dots$, $\mathcal{J} = \{1-\varepsilon, 1, 1+\delta+2\varepsilon, 1, 2-2\varepsilon, 1, 1+\delta+2\varepsilon, \dots\}$, where $\varepsilon = \frac{1}{k}$ and $\delta = r - \lfloor r \rfloor$. An example of this schedule for $r = 5.5$ is depicted on Fig. 3(e).
- For $2i+\sqrt{3} \leq r < 2(i+1)$, $i = 2, 3, \dots$, $\mathcal{J}_R = (1-\varepsilon, 1, 1, \delta, 1, \cdots, 1, \delta, \cdots)$, $\mathcal{J}_R = \{1-\varepsilon, 1, 1, \delta, 1, \cdots, 1, \delta, \cdots\})$, where $\varepsilon = \frac{1}{k}$ and $\delta = r - \lfloor r \rfloor$. An example of this schedule for $r = 5.9$ is depicted on Fig. 3(f).

It can be easily verified that the waiting time achieved by the above jamming sequences are equal to lower bound values stated by the theorem.

■

IV. General schedules

In this section we establish lower and upper bounds on the waiting time of general (non-regular) schedules. Such schedules can include messages of different length, non-periodic schedules, random schedules, etc.

A. Upper bound

In the proof of Theorem 9 we established an upper bound on the additional waiting on the Average Additional Waiting Time $AAWT(M)$ for a message in a regular schedule (see Section III-A for the definition of $AAWT(M)$). This bound, in fact, holds for any type of schedule, which allows us to establish a more general upper bound.

Specifically, let $U(r)$ be the upper bound on $AAWT(M)$ shown in the proof of Theorem 9. Let $\mathcal{S} = \{r_1, r_2, \cdots, r_n\}$, be a general schedule in which message i has length r_i. Let $f(r)$ for $r > 0$ be the frequency of the message of length r in the schedule[1]. Then

$$EWT^{max}(\mathcal{S}) \leq \frac{\sum_{i=1}^{n} f_i r_i U(r_i)}{\sum_{i=1}^{n} f_i r_i} + \frac{1}{2r}.$$

This leads to the following lemma.

Lemma 17: Let $\mathcal{S}$ be a schedule and let r be the expected length of the messages in $\mathcal{S}$. Then the worst case expected waiting time $EWT^{\max}(\mathcal{S})$ of the schedule in the presence of an admissible jammer is bounded by

$$EWT^{\max}(\mathcal{S}) \leq \frac{3}{4} + \frac{11}{4r} \tag{19}$$

Proof: The proof follows form Theorem 9 and the discussion above. ■

B. Lower bound

In this section we establish a lower bound on the worst-case expected waiting time $EWT^{\max}(\mathcal{S})$ of a general schedule $\mathcal{S}$.

Let $\mathcal{J}$ be a jamming sequence in which the length of each jamming pulse and each interval between jamming pulses is exactly one time unit. We divide $\mathcal{J}$ into periods, each period includes a pulse and the (unjammed) interval that separates the preceding and current pulses.

Lemma 18: Let I be a period of $\mathcal{J}$. Then $AWT(\mathcal{J}) \geq \frac{3}{2}$ if there is no update in the time unit that follows the period, and $AWT(\mathcal{J}) \geq \frac{7}{2}$ otherwise.

Proof: We divide I into two parts, I_1 and I_2, such that I_1 includes the first time unit of I and I_2 includes the rest of I. Note that there is no jamming in interval I_1, while I_2 is a jamming pulse.

First, consider the case in which there is no update in the time unit that follows I and there is no update during the subinterval I_2. In this case $AWT(I) = 1 + 1/2 = 3/2$. Indeed, the added waiting time $AWT(I_1)$ is one because each client has to wait exactly one time unit. For the second portion I_2 of I, it holds that $AWT(I_2) = 1/2$ since the clients that belong to I_2 have to wait 0.5 time units, on average.

[1]Frequency function $f(r)$ corresponds to the probability of selecting a message of length r, if each message is equally likely to be selected.

Now, consider the case in which there is an update during I_2. Let $\alpha + 1$ be the distance from the beginning of I to the update. Then

$$AWT(I) \geq \alpha\left(2 - \frac{\alpha}{2}\right) + 1 - \alpha + (1-\alpha)\frac{1-\alpha}{2} =$$
$$= \frac{4\alpha - \alpha^2 + 2 - 2\alpha + 1 + \alpha^2 - 2\alpha}{2} = \frac{3}{2}.$$

Now, we consider the case in which there is an update in the time unit that follows I. We assume that I_1 does not contain an update. Note that this assumption cannot decrease $AWT(I)$. Let α be the distance between the end of I and the update. Then

$$AWT(I) \geq \alpha + (1-\alpha)\left(2 + \alpha + \tfrac{1-\alpha}{2}\right) + \alpha\left(2 + \tfrac{\alpha}{2}\right) + 1 - \alpha =$$
$$= \frac{2\alpha + 5 + \alpha - 5\alpha - \alpha^2 + \alpha^2 + 4\alpha + 2 - 2\alpha}{2} = \frac{7}{2}.$$

■

Lemma 19: Let a server schedule with average length of the messages equal to r be given, and suppose that the fraction of the updates that belong to the non-jammed intervals of $\mathcal{J}$ is p. Then, $EWT(\mathcal{S}, \mathcal{J}) \geq \frac{3}{4} + \frac{2p}{r}$.

Proof: Look at any message M in the server schedule. By Lemma 18, one can see

$$AWT(M) = \begin{cases} \frac{3}{4}r & \textbf{if } \text{the update at the end of M} \\ & \quad \text{belongs to a jammed interval} \\ \frac{3}{4}r + 2 & \textbf{otherwise} \end{cases}$$

Therefore, by taking the average over all messages, we see

$$EWT(\mathcal{S}, \mathcal{J}) = \frac{p\left(\frac{3}{4}r + 2\right) + (1-p)\left(\frac{3}{4}r\right)}{r} =$$
$$= \frac{3}{4} + \frac{2p}{r}.$$

■

Lemma 20: Let $\mathcal{S}$ be a schedule and let r be the expected length of the messages in $\mathcal{S}$. Then, it holds that

$$EWT^{\max}(\mathcal{S}) \geq \frac{3}{4} + \frac{3}{2r}. \tag{20}$$

Proof: Let $\mathcal{J}$ be any regular jamming sequence. The desirable jamming sequence is obtained by modifying schedule $\mathcal{J}$. Specifically, for every update that coincides with the beginning of a jamming pulse of $\mathcal{J}$ we shift the preceding non-jammed interval and compress the following jammed interval by $\varepsilon = \frac{1}{k}$. In the resulting jamming sequence $\mathcal{J}'$ every update happens either during a jamming pulse or during a non-jammed interval. Note also that up to factors of order ε, $\mathcal{J}$ is a regular jamming schedule.

Let $\hat{\mathcal{J}}$ be the jamming sequence which has a jamming pulse during each non-jammed interval of $\mathcal{J}$, and has a non-jamming interval during each jamming pulse of $\mathcal{J}$. By Lemma 19, if $EWT(\mathcal{S}, \mathcal{J}') < \frac{3}{4} + \frac{3}{2r}$ then more than half of the updates fall during a jamming interval in $\mathcal{J}'$. But if an update falls during a jamming interval in $\mathcal{J}'$, then it falls during an non-jamming interval in $\hat{\mathcal{J}}$, thus $EWT(\mathcal{S}, \hat{\mathcal{J}}) \geq \frac{3}{4} + \frac{3}{2r}$. ■

The proof of Theorem 11 follows from Lemmas 20 and 17.

V. CONCLUSION

We investigated the design of efficient anti-jamming schedules for wireless data distribution systems. For such schedules, waiting time and staleness are the key performance parameters. The goal of the jammer is to induce large delays in data transmission and to increase the staleness of the data by forcing the schedule to transmit the data with high level of redundancy.

We focus on combating powerful jammers that have full knowledge about the data distribution system. For such jammers, the standard anti-jamming methods, such as spread-spectrum transmissions are not sufficient in order to guarantee timely delivery of the data, hence additional encoding is required at the packet level.

In this paper we make several contributions. First, we identify optimal and near optimum jamming strategies for the important class of *regular* schedules. In such schedules, the same encoding is used for all packets, which simplifies the design of the mobile device and reduces its cost. Next, we provided lower and upper bounds on the performance of more general class of non-regular schedules. Our results establish a trade-off between the expected waiting time of the client and the staleness of the information in the presence of a jammer.

As a future research, we intend to extend our results to the case in which the broadcast channel is shared by two or more information sources. We also would like to investigate the perfomance of random anti-jamming schedules for wireless data broadcast.

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